AP® Calculus BC
Syllabus 3

AP® Calculus BC Course Outline [C2]

C2—The course teaches all topics associated with Functions, Graphs, and Limits; Derivatives; Integrals; and Polynomial Approximations and Series as delineated in the Calculus BC Topic Outline in the *AP Calculus Course Description*.

Unit I. Limits and Continuity (6 Days—1 Test) [C2]

1. From graphs
2. From tables
3. Symbolic evaluations
4. Limits at infinity
5. Infinite limits
6. Indeterminate forms: 0/0, ∞/∞, ∞−∞, 0 * ∞
7. Graphical look at removable discontinuities
8. Graphical look at nonremovable discontinuities
9. Symbolic consideration of removable discontinuities
10. Symbolic consideration of nonremovable discontinuities

Unit II. Derivatives (21 Days—1 Test) [C2]

1. Average rate of change—related to velocity
2. Average rate of change—related to slope
3. Instantaneous velocity as the average velocity over a smaller time interval
4. Instantaneous velocity as the slope of a curve at a point
5. Local linearity
6. Definition of the derivative as a limit
7. Approximate the derivative at a point graphically
8. Approximate the derivative at a point numerically
9. Determine the graph of the derivative function from the graph of a function
10. Determine the derivative of a function by using the limit definition
11. Explore the relationship between differentiability and continuity

12. Practical meaning of the derivative in a variety of contexts

13. Techniques of differentiation: power rule, product rule, quotient rule

14. Chain rule
    (a) Using Leibniz notation
    (b) Using function notation
    (c) Using parametric equations

15. Implicit functions

16. Inverse functions—using composition (e.g., use $e^{\ln x} = x$ to obtain $d(\ln x)/dx$)

17. Graphical meaning of the second derivative

18. Key theorems relating to continuous functions
    (a) Mean Value Theorem
    (b) Intermediate Value Theorem
    (c) Extreme Value Theorem

Unit III. Applications of the Derivative (21 days—1 Test) [C2]

1. Approximations using the tangent line
2. Related rates
3. Intervals of increase and decrease of a function
4. Intervals of increase and decrease of the derivative—concave up and concave down
5. First derivative test
6. Second derivative test
7. Candidates test
8. Optimization
9. Geometric view of a solution to a differential equation using slope fields
10. Euler's Method to approximate the solution to a differential equation
11. L'Hôpital's rule for cases of 0/0 and ∞/∞
12. Motion on a line: moving left and right, speeding up and slowing down

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13. Relationship of moving right and speeding up to a graph that is increasing and concave up, moving left and slowing down to decreasing and concave up, etc.

Unit IV. Integration and Antidifferentiation (6 Days—1 Test) [C2]

1. Variety of examples of summing to approximate total change given tabular data
2. Concept of a Riemann sum
3. Definite Integral defined as the limit of a Riemann sum
4. Link between the definite integral and area—advantages and pitfalls
5. Properties of the definite integral
6. Antidifferentiation motivated by finding a position function from a velocity function
7. Fundamental theorem of calculus motivated by finding distance traveled two different ways

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Unit V. Numerical Approximations of a Definite Integral (5 Days—No Test) [C2]

1. Riemann sums—left, right, midpoint
2. Trapezoid rule
3. Simpson's rule
4. Relationship between Trapezoid, Midpoint, and Simpson's rules
5. Investigation as to how each of these techniques improves if the number of subdivisions is doubled, tripled, or multiplied by a factor of k

Unit VI. Techniques of Antidifferentiation (11 Days—1 Test) [C2]

1. From known derivatives
2. From a graph of a derivative
3. Simple substitution—form completion
4. Substitution—actual substitution needs to be made, including trig substitution
5. Parts
6. Improper Integrals

Unit VII. Applications of Definite Integral and Antidifferentiation (32 Days—2 Tests) [C2]

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1. Determine specific antiderivatives using initial conditions
2. Solution to separable differential equations with and without initial conditions
3. Writing a differential equation to translate a verbal description
4. Partial fractions in the context of the logistic equation
5. Representation of a particular antiderivative by using the Fundamental Theorem of Calculus
6. Analysis of functions defined by a definite integral
7. Area, including regions bounded by polar curves
8. Average value of a function
9. Distance as the definite integral of speed
10. Length of a curve, including polar and parametric curves
11. Work
12. Variety of other problems using the integral of a rate of change to determine total or accumulated change
13. Variety of other problems where the emphasis is on setting up a Riemann sum and taking its limit

Unit VIII. Series (38 Days—3 Tests) [C2]

1. Infinite series defined as the limit of a sequence of partial sums
2. Series of constants
   (a) Geometric series
   (b) Harmonic series, P-series
   (c) Alternating series
3. Tests for convergence
   (a) Integral
   (b) Comparison
   (c) Limit comparison
   (d) Ratio test—thought of as eventually geometric

4. Power series
   (a) Taylor polynomials as approximations for functions
   (b) Taylor series centered at $x = a$
   (c) Use of known Maclaurin series for e^x, $\sin x$, $1/(x+1)$, $(1+x)^p$ to form new series
   (d) Differentiation and antidifferentiation of series to determine new series
   (e) Functions defined by power series
   (f) Interval and radius of convergence
   (g) Error bounds
      i convergent geometric series
      ii using integral test
      iii convergent alternating series
      iv Lagrange

Teaching Strategies

1. Each topic is presented numerically, geometrically, symbolically, and verbally as students learn to communicate the connections among these representations. **[C3]**

2. Justifications of responses and solutions are part of the routine when solving problems. Students are encouraged to express their ideas in carefully written sentences that validate their process and conclusions. **[C4]**

3. Students make extensive use of the TI-83 calculator. Each student has his or her own calculator.

4. Students use programs in their calculators to:
   a. Investigate limits of functions
   b. Confirm characteristics (e.g., concavity) of graphs of functions
   c. Perform numerical integration
   d. Find points of inflection
   e. Show Riemann sums
   f. Compute partial sums

C3—The course provides students with the opportunity to work with functions represented in a variety of ways—graphically, numerically, analytically, and verbally—and emphasizes the connections among these representations.

C4—The course teaches students how to communicate mathematics and explain solutions to problems both verbally and in written sentences.

g. Use Euler's method

h. Show a slope field

i. Draw a solution curve on a slope field

j. To sketch implicitly defined functions **[C5]**

C5—The course teaches students how to use graphing calculators to help solve problems, experiment, interpret results, and support conclusions.

5. From the middle of October throughout the rest of the year, students are assigned three free-response questions from AP Released Exams every three days. These questions are graded as they would be at an AP Reading. Students may use a calculator for any question for which a calculator was allowed when the question appeared on the exam, and they may not use a calculator for any question for which a calculator was not allowed when the question appeared on the exam.

6. Each week students have one or more surprise quizzes containing five multiple-choice items from AP Released Exams. Calculator usage is the same as described above.

7. All tests contain material from previous units. Students are responsible for all material covered to the date of the test. All tests are two periods in length, one with calculator usage and one without.

8. Students are encouraged to work cooperatively on in-class worksheets, graded AP problems, and take-home exams. **[C4]**

C4—The course teaches students how to communicate mathematics and explain solutions to problems both verbally and in written sentences.

9. Circular functions, exponential functions, and logarithmic functions are used throughout the course. Students have previously studied these functions, so we deal with the derivatives of these functions early in the course.

10. Students learn to use the spreadsheet program Excel.

11. Excel is used for Euler's method and for summing examples.

References and Materials

Major Texts

Anton, Howard. *Calculus*. 3rd ed./brief ed. New York: John Wiley & Sons, 1988.

Finney, Ross L., Franklin Demana, Bert Waits, and Daniel Kennedy. *Calculus—Graphical, Numerical, Algebraic*. 2nd ed. Addison-Wesley Longman, 1999.

Hughes-Hallett, Deborah, Andrew M. Gleason, et al. *Calculus*. 2nd ed. New York: John Wiley & Sons, 1998.